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MISCELLANEOUS

Recombination Accompanied by Teleportation of "Quantum Entanglement"

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Abstract—It is remarked that the recently proposed [2] and partially realized [3] teleportation of quantum states of microparticles is (in a certain modification) a natural element of the process of cross recombination of radical spin pairs [4]. In the present paper the situation where singlet and triplet pairs are created in the medium and then diffuse and can recombine, if the particles encountering one another are in a singlet state, is examined in detail. As a result of cross recombination and teleportation, there arises a countable set of spin states of radical pairs. The distributions of the densities of radical pairs corresponding to this set are found under stationary and spatially uniform conditions. © *2000 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

The phenomenon of so-called quantum entanglement (QE) has been known for a long time, but it has aroused great interest in the last few years in connection with its role in the quantum theory of information and the theory of a quantum computer (see, for example, [1]). Quantum entanglement gives rise to specific correlations, which have no classical analog, between subsystems of a single quantum system. In the present paper the following general negative definition of QE is used: A state (the density matrix) $\hat{\rho}(1, 2)$ of the union of two subsystems S_1 and S_2 is said to be "nonentangled" if it can be represented as a convex linear combination of products of the states of the subsystems, specifically,

$$\hat{\rho}(1, 2) = \sum_i p_i \hat{\rho}_i(1) \otimes \hat{\rho}_i(2), \tag{1}$$

where $p_i > 0$ and $\sum_i p_i = 1$. We note that for a "pure" state $\hat{\rho}(1, 2) = |\psi(1, 2)\rangle\langle\psi(1, 2)|$ this definition of "nonentanglement" reduces to the well-known requirement that the wave function be factorizable: $|\psi(1, 2)\rangle = |\psi(1)\rangle \otimes |\psi(2)\rangle$.

The existence of QE is the main attribute in realizing the so-called quantum teleportation [2, 3]. We shall be interested in a modification of this phenomenon, specifically, quantum entanglement teleportation (QET). The crux of the QET process (in the form in which it appears below) is as follows. Two individuals, Alice and Bob, each possess a pair of spin-1/2 particles. We denote Alice's particles by the numbers 1 and 3 and Bob's particles by the numbers 2 and 4. We consider a situation where the state of all four particles has the form

$$\hat{\rho}(1, 2, 3, 4) = \hat{\rho}(1, 2) \otimes \hat{\rho}(3, 4). \tag{2}$$

In this case, as one can easily see, the pairs of Alice's and Bob's particles *a priori* are in the states

$$\begin{aligned}\hat{\rho}^{(pre)}(1, 3) &= \mathrm{Tr}_{2,4}\hat{\rho}(1, 2, 3, 4) = \hat{\rho}(1) \otimes \hat{\rho}(3),\\ \hat{\rho}^{(pre)}(2, 4) &= \mathrm{Tr}_{1,3}\hat{\rho}(1, 2, 3, 4) = \hat{\rho}(2) \otimes \hat{\rho}(4)\end{aligned} \tag{3}$$

and in accordance with equation (1) they do not possess the property of QE. Alice performs a measurement on her particles, checking to see whether or not her pair is in a singlet state,

$$\hat{P}_0(1, 3) \equiv |\Psi_0(1, 3)\rangle\langle\Psi_0(1, 3)|, \tag{4}$$

where

$$|\Psi_0(1, 3)\rangle = \frac{1}{\sqrt{2}}(|\psi_+(1)\rangle \otimes |\psi_-(3)\rangle - |\psi_-(1)\rangle \otimes |\psi_+(3)\rangle),$$

$|\psi_\pm(i)\rangle$ is the state of particle i with spin projection $\pm 1/2$ in a certain fixed direction. We note that the state $\hat{P}_0(1, 3)$ refers to the so-called "maximally entangled" state, in

which the states of the particles 1 and 3 are maximally mixed:

$$\hat{\rho}_0(1) = \mathrm{Tr}_3\hat{P}_0(1,3) = \frac{1}{2}\hat{I}(1),$$

$$\hat{\rho}_0(3) = \mathrm{Tr}_1\hat{P}_0(1,3) = \frac{1}{2}\hat{I}(3).$$

All information carried by such states is contained in the interparticle correlations.

For a positive outcome of an experiment, Alice's pair at Bob's location are in the state

$$\hat{\rho}^{(post)}(2,4) = \frac{\mathrm{Tr}_{1,3}[\hat{P}_0(1,3)(\hat{\rho}(1,2)\otimes\hat{\rho}(3,4))\hat{P}_0(1,3)]}{\mathrm{Tr}_{1,2,3,4}[\hat{P}_0(1,3)(\hat{\rho}(1,2)\otimes\hat{\rho}(3,4))\hat{P}_0(1,3)]} \quad (5)$$

in accordance with von Neumann's projection postulate. It is convenient to introduce the QET mapping $\tau_{1,3}$, acting on the set of all states of particles 1, 2, 3, and 4, of the form (2). The result of this mapping is the state (5):

$$\tau_{1,3}\colon \hat{\rho}(1,2)\otimes\hat{\rho}(3,4) \longrightarrow \hat{\rho}^{(post)}(2,4). \quad (6)$$

In the next section it would be shown that in the presence of an initial "entanglement" within the pairs 1, 2 and 3, 4 (the so-called QE resource) the state $\hat{\rho}^{(post)}(2,4)$, in contrast to $\hat{\rho}^{(pre)}(2,4)$, is "entangled."

The fundamental condition, anticipating any use of quantum correlations of the QE type which exist in the state $\hat{\rho}^{(post)}(2,4)$, is Bob receiving a signal from Alice indicating a positive outcome for her experiment. In the literature, this circumstance is often interpreted as the necessity of the existence of two types of channels for transmitting information in realizing QET (and also standard quantum teleportation)—a classical channel, along which Alice reports about the outcome of her experiment, and the so-called Einstein–Podolsky–Rosen (EPR) quantum channels, formed by the world lines of "entangled" pairs of particles 1, 2 and 3, 4. It should be noted that in our exposition the states of the pairs 1, 2 and 3, 4 appear as two equivalent EPR channels. The pertinence of such a point of view is dictated by the specific nature of the problem considered below. Our symmetric interpretation differs from the conventional one [2], where the EPR channel is only one pair, for example, 1, 2, and this channel is used for teleportation of the "entanglement" existing between the spin states of particles 3 and 4 into a new pair 2, 4. A more accurate term, used in the literature for the process under study, *entanglement swapping*, is the transport or transfer of entanglement.

The view that QET is an exotic phenomenon is probably incorrect. In the present paper we study a simple model process of creation and recombination of spin radicals in a medium that is accompanied by QET. As far as I know, the aspect associated with teleportation was first brought up in [4]. The crux of the phenomenon is as follows. Let us assume that only singlet pairs can be created and recombined, and the spin states of the correlated pairs are not subjected to any perturbations and remain singlets in the intervals between the creation and recombination events. As a result of diffusion, particles from different pairs can approach one another.[1] Successful recombination is identical to a positive outcome of Alice's experiment. The photon emitted on recombination carries a classical message indicating success, and the remaining fragments of the two pairs are in a singlet state. The kinetics of this process, taking account of possible creation of triplet pairs also, is studied in Section 3, but first we need to develop a convenient language for quantitative description of QET in our model.

2. GENERAL RELATIONS FOR QET ACCOMPANYING RECOMBINATION

As already mentioned in the Introduction, QET in the presence of recombination appears to be especially simple if all correlated pairs are singlets. In this case, each recombination event annihilates two singlet pairs and creates one new pair—"teleportation of singletness" occurs. The problem of this section is to determine the quantitative characteristics of QET accompanying recombination of fragments of two pairs mixed in an arbitrary manner. These characteristics are required in the situation where triplet pairs appear in the medium for some reason. As will be shown below, this is equivalent to a decrease in the quality of EPR channels. We shall confine our attention to the spatially isotropic situation. In this case, the most general states of pairs of particles *i* and *j* have the form

$$\hat{\rho}^{(\alpha)}(i,j) = \frac{3\alpha+1}{4}\hat{P}_0(i,j) + \frac{1-\alpha}{4}(\hat{P}_{1,-1}(i,j) + \hat{P}_{1,0}(i,j) + \hat{P}_{1,+1}(i,j)), \quad (7)$$

where we have introduced the parameter $\alpha \in [-1/3, 1]$, and the operators $\hat{P}_{1,-1}(i,j)$, $\hat{P}_{1,0}(i,j)$, and $\hat{P}_{1,+1}(i,j)$, by analogy with equation (4), are projectors onto the states

$$|\Psi_{1,-1}(i,j)\rangle = |\psi_-(i)\rangle\otimes|\psi_-(j)\rangle,$$

[1] This fundamentally distinguishes the model under consideration from the process of geminal (cell) recombination [5]. The recombination of radicals from different pairs has been studied, for example, in [6].

$$|\Psi_{1,0}(i,j)\rangle = \frac{|\psi_+(i)\rangle \otimes |\psi_-(j)\rangle + |\psi_-(i)\rangle \otimes |\psi_+(j)\rangle}{\sqrt{2}},$$

$$|\Psi_{1,+1}(i,j)\rangle = |\psi_+(i)\rangle \otimes |\psi_+(j)\rangle,$$

respectively. The sum in parentheses in equation (7) is, to within a factor of 1/3, an isotropic triplet state. The parameter α, which gives the fraction of singletness and tripletness, is introduced in a manner so that $\alpha = 1$ corresponds to a purely singlet pair, $\alpha = 0$ corresponds to an uncorrelated maximally mixed state of the particles i and j, and $\alpha = -1/3$ corresponds to a purely triplet state.

It is not difficult to show that in the model under consideration the recombination of two particles from different pairs occurs with probability 1/4 (this immediately determines the normalization factor in equation (5)) and, which is especially important, it dictates the unique multiplicative character of the teleportation mapping from equation (6):

$$\tau_{1,3}: \hat{\rho}^{(\alpha)}(1,2) \otimes \hat{\rho}^{(\beta)}(3,4) \longrightarrow \hat{\rho}^{(\alpha\beta)}(2,4). \quad (8)$$

Since the parameters α and β of the initial pairs do not exceed 1 in magnitude, we have

$$0 \le |\alpha\beta| \le \min(|\alpha|, |\beta|).$$

This reflects the unavoidable degradation of the teleported state as compared with the states of the EPR channels 1–2, 3–4.

If it is assumed that triplet pairs, just like singlet pairs, arise as a result of a dissociation-type process, then it can be concluded from the multiplicative character of the mapping τ that only pairs whose parameter α is an integer power $n = 0, 1, 2, \ldots$ of the number $-1/3$, corresponding to a triplet state, are present in the medium:

$$\hat{\rho}^{(n)}(i,j) = \frac{1}{4}\left[1 + 3\left(-\frac{1}{3}\right)^n\right]\hat{\rho}^{(0)}(i,j) + \frac{3}{4}\left[1 - \left(-\frac{1}{3}\right)^n\right]\hat{\rho}^{(1)}(i,j). \quad (9)$$

Here the state $\hat{\rho}$ is indexed by the power n and not by the parameter α itself, as in equation (7). In the chosen notation, $\hat{\rho}^{(0)}(i,j)$ corresponds to a singlet state and $\hat{\rho}^{(1)}(i,j)$ corresponds to a triplet state. In the new notation the teleportation mapping becomes additive:

$$\tau_{1,3}: \hat{\rho}^{(n)}(1,2) \otimes \hat{\rho}^{(m)}(3,4) \longrightarrow \hat{\rho}^{(n+m)}(2,4). \quad (10)$$

The maximally mixed state must now be designated as $\hat{\rho}^{(\infty)}(i,j)$.

3. KINETIC EQUATIONS AND THEIR SOLUTIONS

We shall now formulate the kinetic model. We shall consider an unbounded medium in which creation and recombination of pairs of particles with spin 1/2 occur.[2] For simplicity, we shall assume that the particles created are identical. We assume that only singlet pairs can recombine, while both singlet and triplet pairs can be created. In the intervals between creation and recombination events, the particles diffuse with diffusion coefficient D, which does not depend on the state of the particle. We introduce the set of functions $f^{(n)}(\mathbf{r}_1, \mathbf{r}_2)$, two-coordinate spatial densities of pairs of particles in the spin state $\hat{\rho}^{(n)}$ from equation (9). The densities satisfy the following system of kinetic equations:

$$\begin{aligned}\partial_t f^{(n)}(\mathbf{r}_1,\mathbf{r}_2) &= -\nu f(\mathbf{r}_1) f^{(n)}(\mathbf{r}_1,\mathbf{r}_2) - \nu f^{(n)}(\mathbf{r}_1,\mathbf{r}_2) f(\mathbf{r}_2) \\ &+ \nu \sum_{k=0}^{n} \int f^{(k)}(\mathbf{r}_1,\mathbf{r}') f^{(n-k)}(\mathbf{r}',\mathbf{r}_2) d^3r' \\ &+ D(\Delta_{\mathbf{r}_1} + \Delta_{\mathbf{r}_2}) f^{(n)}(\mathbf{r}_1,\mathbf{r}_2) + \gamma^{(n)}(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_2).\end{aligned} \quad (11)$$

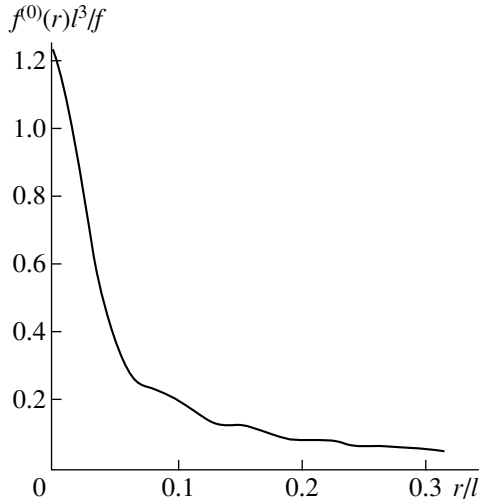
Here the first two terms on the right-hand side correspond to annihilation of pairs accompanying recombination of one of its particles, located at the points $\mathbf{r}_1$ and $\mathbf{r}_2$;

$$f(\mathbf{r}) = \sum_{n=0}^{\infty} \int f^{(n)}(\mathbf{r},\mathbf{r}') d^3r'$$

is the total density of the recombining pairs. The third term corresponds to creation of a pair accompanying annihilation of two other pairs. The parameters of the created pair are related with the parameters of the annihilated pairs via the QET mapping (10). The last two terms are diffusion terms and the source of local pair creation. We assume that only the quantities $\gamma^{(0)}$ and $\gamma^{(1)}$ are different from zero. We note that in the case of singlet and triplet pairs we use singular sources in equation (11). At the same time, it is known that because of the statistics of spin-1/2 particles the coordinate density matrix of a created triplet pair and together with it the function $\gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2)$ must approach zero as $\mathbf{r}_1 \longrightarrow \mathbf{r}_2$. However, in our approach, where all spatial scales are much greater than the molecular scale (and this is the characteristic size of the radical pair just created), we are fully justified in replacing the true source of triplet pairs by its singular approximation.

We shall consider the spatially uniform and stationary situation where $f^{(n)}(\mathbf{r}_1, \mathbf{r}_2) = f^{(n)}(|\mathbf{r}_1 - \mathbf{r}_2|)$, $f(\mathbf{r}) = f$,

[2] Spin must be understood in the generalized sense. It can be the standard spin and any dichotomous observable, for example, chirality. In the latter case, the states "spin up" and "spin down" must be identified with right-hand and left-hand enantiomers.



Stationary density of singlet pairs (see explanation in the text).

$\gamma^{(n)}(\mathbf{r}) = \gamma^{(n)}$. Then the equations for the Fourier transforms of the functions $f^{(n)}(|\mathbf{r}_1 - \mathbf{r}_2|)$,

$$f_q^{(n)} = \int f^{(n)}(|\mathbf{r}|)\exp(-i\mathbf{q}\cdot\mathbf{r})d^3r,$$

acquire the especially simple form

$$2(\nu f + Dq^2) f_q^{(n)} = \nu \sum_{k=0}^{n} f_q^{(k)} f_q^{(n-k)} + \gamma^{(n)}. \qquad (12)$$

The total stationary particle density $f = \sum_{n=0}^{\infty} f_0^{(n)}$ is determined by the same system of equations:

$$f = \sqrt{\frac{\gamma^{(0)}}{\nu} + \frac{\gamma^{(1)}}{\nu}}. \qquad (13)$$

A quadratic equation is obtained for the density $f_q^{(0)}$ of singlet pairs (and only for $f_q^{(0)}$). The stable solution is

$$f_q^{(0)} = f + \frac{D}{\nu}q^2 - \sqrt{\left(f + \frac{D}{\nu}q^2\right)^2 - \frac{\gamma^{(0)}}{\nu}}. \qquad (14)$$

The solution of the system (12) for the Fourier transforms $f_q^{(n)}$ of the densities for $n = 1, 2, \ldots$ has the form

$$f_q^{(n)} = C_n\left(\frac{\gamma^{(1)}}{2\nu}\right)^n\left[\left(f + \frac{D}{\nu}q^2\right)^2 - \frac{\gamma^{(0)}}{\nu}\right]^{1/2-n}, \qquad (15)$$

where the constants C_n are determined from the system of recurrence relations

$$C_n = \frac{1}{2}\sum_{k=1}^{n-1} C_k C_{n-k},$$

$C_1 = 1$, $n = 2, 3, \ldots$. The characteristic form of the distribution $f^{(0)}(r)$ is displayed in the figure.

The plot was constructed for equal rates of creation of triple and singlet pairs ($\gamma^{(0)} = \gamma^{(1)}$). An expression for the characteristic length l is presented in the Conclusions (see equation (16)). We note the unique step character of the function $f^{(0)}(r)$.

4. CONCLUSIONS

We have described on the basis of a simple "mass action" type law the kinetics of the creation, diffusion, and recombination of pairs of particles in "entangled" quantum spin states. A nontrivial feature of the model is that the process of QET in the recombination of particles is taken into account. The stationary uniform regime admits, as shown above, a simple analytical description. We note that taking account of the evolution of the spin states between creation and recombination events should substantially change the results obtained.

As follows from equations (12) and the figure, as a result of diffusion and the QET process, a substantial fraction of "entangled" spin states with characteristic size $0.1l$ can appear, where

$$l = \left(\frac{D^2}{\nu(\gamma^{(0)} + \gamma^{(1)})}\right)^{1/4}. \qquad (16)$$

Nonlocal, nonclassical spin correlations arise in the medium. This should have nontrivial thermodynamic consequences. We shall examine as an example the fluctuations of the total projection of the spin of the particles located in a certain region Ω. For simplicity we shall assume that only singlet pairs are present. We present without derivation the result of calculation of the distribution function $p(m)$ for the probability of finding the total spin projection m of all particles in a chosen direction:

$$p(m) = I_{|2m|}(\xi)e^{-\xi}. \qquad (17)$$

Here $I_n(z)$ is a modified Bessel function, and the parameter

$$\xi = \int_{\Omega} d^3r_1 \int_{\bar{\Omega}} d^3r_2 f^{(0)}(|\mathbf{r}_1 - \mathbf{r}_2|) \qquad (18)$$

is defined as the integral over the volume of the region Ω and its exterior part $\bar{\Omega}$. Physically, the parameter ξ is the average number of singlet pairs, one fragment of which lies inside Ω and the other outside Ω. It is obvious that only such pairs will contribute to the total spin of the region Ω. It is also obvious that ξ depends not only on the volume but also on the shape of the

region Ω, especially if the characteristic size of Ω or its individual parts is of the order of l.

The situation where a source of triplet pairs is present is more complicated. This case, and the general problems of thermodynamics of a medium of spin-correlated pairs, require a separate analysis.

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